Two new sum rules for octet-baryon magnetic moments (μ) and constraints on QCD sum rules from new experimental determination of μ-s for the decuplet.

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Abstract :

Recently the $\mu_{\Delta^{++}}$ was found from a fit to $\pi^+ p$ scattering [1]. This enable us to pinpoint condensate parameters more precisely in the context of QCD sum rules (QCDSR).

In the octet sector, the Coleman-Glashow sum rule (CGSR) [2] is violated by the experimental μ-s. QCDSR allows us to write down two sum rules similar to the CGSR, which are obeyed by the experimental magnetic moments, whereas they rule out a specific model using the Wilson loop approach and a particular chiral quark model.

It is amusing to note that the QCDSR allows us to write down the quark and gluon condensates in terms of measurables like the μ-s of the nucleons and the $\Sigma^{\pm}$.

Keywords : QCD sum rules, magnetic moments of octet and decuplet baryons, magnetic moment sum rules.

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1 Introduction

Magnetic moments of baryons depend very sensitively on model parameters. So accurately measured values of baryon magnetic moments are very useful to constrain the validity of modeling.

In the decuplet sector μ_Ω was measured accurately and differs from most of the theoretical estimates, thus posing a challenge to the latter. It was shown [3] that this can be explained from QCDSR and QCD condensate parameters are thereby constrained.

Recently there has been much experimental and theoretical studies, seemingly a little isolated, with different groups not conscious of each other's work. But these studies of magnetic moments can be correlated to evolve a picture of the QCD vacuum which is very rich. Correlations between μ_B should also be interesting to experimentalists. Thus for example Kotulla et al. [4], during their determination of μ_{Δ^+} should have found the QCDSR relation $\mu_{\Delta^{++}} = \frac{1}{2}\mu_{\Delta^+}$ interesting, in view of the earlier determination of $\mu_{\Delta^{++}}$ by Castro and Mariano [1]. We write down two other sum rules involving octet baryon μ_B, hoping to stimulate more studies of these objects.

QCDSR enables us to write down the quark and gluon condensates in terms of the octet magnetic moments, for example μ_p, μ_n and $\mu_{\Sigma^\pm}$.

We also find that the magnetic susceptibility needs to be very large to fit the determination of the magnetic moment of Δ^{++}, made in [1], from the most sensitive observables in radiative $\pi^+ p$ scattering.

Iqubal et al. [3] used the QCDSR to fit the Ω^- magnetic moment. μ_{Ω^-} has been the subject of many studies [5, 6, 7, 8, 9, 10]. The magnetic moment was unknown, when the large colour Fock approximation paper [5] was published. But on hindsight, the value predicted there, within the acceptable parameter range, agrees with the presently determined experimental result [11][1]. The results of Lee [6] using QCDSR and those from the lattice calculation [8] underestimated it whereas the light- cone relativistic quark model [9] and the chiral quark soliton model [10] overestimated it. This intriguing situation was investigated by looking at the calculations of Lee using a slightly different point of view advocated in [14] and it was found that one indeed gets good agreement with experiment [3].

Further, it was pointed out in [6, 3] the $\mu_{\Delta^{++}}$ depend sensitively on the magnetic susceptibility. This moment is now obtained in [1]. They have determined the μ of the Δ^{++} resonance by using a full dynamical model which consistently describes the

[1]The methods of this calculation are now used for strange star matter [12, 13]

elastic and radiative π^+ p scattering data. It also reproduce very well the total and differential cross-sections for elastic π^+ p scattering close to the resonance region. It provides an amplitude for radiative π^+ p scattering that satisfies electro magnetic gauge invariance when finite width effects of Δ^{++} resonance are taken into account. From their determination we can fix the magnetic susceptibility parameter of QCDSR.

As already mentioned, very recently Kotulla et al. have investigated the reaction $\gamma p \rightarrow \pi^0 \gamma\prime p$. Through the reaction channel they arrived at the magnetic dipole moment of the Δ^+ (1232) resonance [4]. Their measured value is also consistent with QCDSR.

Table 1: The experimental values of magnetic moments in unit of μ_N

p	n	Ξ^-	Ξ^0	Σ^+	Σ^-	Ω^-	Δ^+	Δ^{++}
2.793	-1.913	-0.6507	-1.25	2.458	-1.16	2.019	$2.7^{+2.5}_{-2.8}$	6.14 ± 0.51

We have summarized the values of experimentally determined magnetic moments [4, 1, 11] in the Table 1.

The Coleman and Glashow sum rule CGSR [2] is given by

$$\Delta CG \ = \ \mu_p \ - \ \mu_n \ + \ \mu_{\Sigma^-} \ + \ \mu_{\Sigma^+} \ + \ \mu_{\Xi^0} \ - \ \mu_{\Xi^-} \ = \ 0 \tag{1}$$

Experimental numbers give $\Delta CG \ = \ 0.49 \ \mu_N$.

From the experimental values of octet magnetic moments we can get the values of the quark and gluon condensates respectively :

$$a \ = \ -2\pi^2 < \bar{q}q > \ = \ \sqrt{-0.4618(\mu_p \ + \ 2 \ \mu_n) \ - \ 1.8382(\mu_{\Sigma^+} \ + \ 2 \ \mu_{\Sigma^-})} \tag{2}$$

$$b \ = \ < g_s^2 G^2 > \ = \ -4.4545(\mu_p \ + \ 2 \ \mu_n) \ - \ 21.2651(\mu_{\Sigma^+} \ + \ 2 \ \mu_{\Sigma^-}) \tag{3}$$

Putting the values of the experimental moments one gets numerical values $a = 0.472$ and $b = 1.667$. The former matches with the value we use, the latter differs in the last figure, we use 1.664.

We have two new sum rules, SR1 and SR2, resulting from the scaling of the baryonic coupling to its current [14]. These are as follows:

$$\Delta SR1 \ = \ (\mu_p \ + \ 2\mu_n) \ + 6.7096(\mu_{\Sigma^+} \ + \ 2\mu_{\Sigma^-}) \ - \ 3.4484(\mu_{\Xi^-} \ - \ \mu_{\Xi^0}) \ + 2.1741 \ = \ 0 \tag{4}$$

$$\Delta SR2 = (\mu_p + 2\mu_n) + 4.7738(\mu_{\Sigma^+} + 2\mu_{\Sigma^-}) - 0.9988(\mu_{\Xi^-} - \mu_{\Xi^0}) + 0.9781 = 0 \quad (5)$$

Using the experimental values of magnetic moments [11] the left hand side of these two sum rules (Eq. 4 and Eq. 5) give $\Delta SR1 = 4.4929 \times 10^{-4} \mu_N$ and $\Delta SR2 = 5.3175 \times 10^{-3} \mu_N$. These sum rules are very powerful. For example the chiral quark model for octet baryon magnetic moments of Dahiya and Gupta [15] becomes questionable, although it satisfies the ΔCG while fitting the experimental moments approximately. The agreement to ΔCG obtained in this paper is clearly accidental; the small departures from the experimental moments cancel for CGSR, but do not for $\Delta SR1$ and $\Delta SR2$ (see Table 2). However it is possible that with more judicious choice of parameters the chiral quark

Table 2: The values of magnetic moments and sum rules in unit of μ_N for [16] and [15]. Note that the values of ΔSR should be zero.

p	n	Ξ^-	Ξ^0	Σ^+	Σ^-	$\Delta SR1$	$\Delta SR2$	ΔCG	ref
2.744	-1.955	-0.598	-1.278	2.461	-1.069	0.830	0.675	0.489	[16]
2.800	-1.990	-0.560	-1.240	1.430	-1.200	1.148	0.739	0.480	[15]

model may be able to satisfy the new sum rules given by us.

The same comments apply to the model of Ha and Durand [16] in Table 2. They fit the ΔCG fairly well but their model fails for $\Delta SR1$ and $\Delta SR2$. The decomposition of the magnetic moments in terms of the parameters of Table VI of their paper may perhaps be used effectively to satisfy the new sum rules.

2 QCDSR for decuplet μ_B.

As is widely known, QCDSR is a very powerful tool in revealing a deep connection between hadron phenomenology and vacuum structure [17] via a few condensates like a, b, related to the quark (q) and gluon (G) vacuum expectation values. These can be used for evaluating μ_B [18, 19], where some new parameters enter, for example, χ, κ and ξ, defined through the following equations :

$$< \bar{q}\sigma_{\mu\nu}q >_F = e_q \chi < \bar{q}q > F_{\mu\nu}, \quad (6)$$

$$< \bar{q}gG_{\mu\nu}q >_F = e_q \kappa < \bar{q}q > F_{\mu\nu}, \quad (7)$$

$$< \bar{q}\epsilon_{\mu\nu\rho\gamma}G^{\rho\gamma}\gamma_5 q >_F = e_q \xi < \bar{q}q > F_{\mu\nu}. \quad (8)$$

where the F denotes the usual external electromagnetic field tensor. Lee [6] very carefully evaluated the contributions of these operators to the magnetic moments of the Ω^- and Δ^{++}, the latter emerging from the former when the quark mass m_s, is put equal to zero, the parameter f and ϕ are put equal to 1 and the quark charge $e_s = -1/3$ is replaced by $e_u = 2/3$. The parameter f and ϕ measure the ratio of values for quark condensates and quark spin-condensates with strange and (ud) quarks.

$$f = \frac{<\bar{s}s>}{<\bar{u}u>}, \tag{9}$$

$$\phi = \frac{<\bar{s}\sigma_{\mu\nu}s>}{<\bar{u}\sigma_{\mu\nu}u>} \tag{10}$$

For the expression for the μ_{Ω^-} and Δ^{++} sum rules we refer the expressions derived in Lee [6] which we reproduce here for the sake of completeness, in terms of the Borel parameter M and the intermediate state contribution A :

$$\begin{aligned}
&\frac{9}{28}e_s L^{4/27}E_1 M^4 - \frac{15}{7}e_s f\phi m_s \chi a L^{-12/27}E_0 M^2 + \frac{3}{56}e_s b L^{4/27} - \frac{18}{7}e_s f m_s a L^{4/27} \\
&-\frac{9}{28}e_s f\phi(2\kappa+\xi)m_s a L^{4/27} - \frac{6}{7}e_s f^2\phi\chi a^2 L^{12/27} - \frac{4}{7}e_s f^2\kappa_v a^2 L^{28/27}\frac{1}{M^2} \\
&-\frac{1}{14}e_s f^2\phi(4\kappa+\xi)a^2 L^{28/27}\frac{1}{M^2} + \frac{1}{4}e_s f^2\phi\chi m_0^2 a^2 L^{-2/27}\frac{1}{M^2} \\
&-\frac{9}{28}e_s f m_s m_0^2 a L^{-10/27}\frac{1}{M^2} + \frac{1}{12}e_s f^2 m_0^2 a^2 L^{14/27}\frac{1}{M^4} \\
&= \tilde{\lambda}_\Omega^2\left(\frac{\mu_\Omega}{M^2} + A\right)e^{-M_\Omega^2/M^2}.
\end{aligned} \tag{11}$$

Here

$$E_n(x) = 1 - e^{-x}\sum_n \frac{x^n}{n!},\ x = w_B^2/M_B^2 \tag{12}$$

where w_B is the continuum, and

$$L = \frac{ln(M^2/\Lambda_{QCD}^2)}{ln(\mu^2/\Lambda_{QCD}^2)} \tag{13}$$

For evaluating the magnetic moment we use the above equation and divide by the equation for the mass sum rule given earlier by Lee [20]. Thus we eliminate the parameter λ_{Ω^-} in the spirit of [14] and we get an excellent fit to the resulting numbers in the form $\mu_{\Omega^-} + A/M^2$. We find that the results are not very sensitive to κ_v, the so called factorization violation parameter, defined through

$$<\bar{u}u\bar{u}u> = \kappa_v <\bar{u}u>^2 . \tag{14}$$

Neither are the results very sensitive to the parameters κ and ξ. We use the crucial parameters a and b from [14], since they must fit the octet baryon moment-differences $(\mu_p - \mu_n)$ and $(\mu_{\Sigma^+} - \mu_{\Sigma^-})$. It was shown in [14] that by using the empirical scaling of the $\tilde{\lambda}$ with the $(baryon\ mass)^3$ - these differences depend only of a and b, and one gets $a = 0.475\ GeV^3$ and $b = 1.695\ GeV^4$. In this paper we have used slightly different values 0.472 and 1.664 for a and b. Further, to fit the difference $(\mu_{\Xi^0} - \mu_{\Xi^-})$, m_s was set to be 170 MeV in [14] and we use this value.

Table 3: The values of parameters and their corresponding magnetic moments

χ		μ_{Ω^-}			$\mu_{\Delta^{++}}$		
	$\xi\ =$	-1	-2	-3	-1	-2	-3
11.0		-1.945	-1.955	-1.966	5.84	5.87	5.90
11.1		-1.956	-1.966	-1.977	5.89	5.92	5.95
11.2		-1.967	-1.977	-1.988	5.94	5.97	5.99
11.3		-1.978	-1.988	-1.998	5.99	6.02	6.05
11.4		-1.988	-1.999	-2.009	6.04	6.07	6.09
11.5		-1.999	-2.010	-2.020	6.09	6.11	6.14
11.6		-2.010	-2.020	-2.031	6.14	6.16	6.19
11.7		-2.021	-2.032	-2.042	6.18	6.21	6.24
11.8		-2.032	-2.042	-2.053	6.23	6.26	6.29

Table 3 shows the dependence of the magnetic moments on the parameters. Clearly the agreement with experiment is very good both for $\mu_{\Delta^{++}}$ and μ_{Ω^-}. Obviously the former does not depend on f and ϕ. It is found that $\chi \sim 11$ is the best choice for the $\mu_{\Delta^{++}}$. For such a χ one should take $\phi \sim 0.35$ and $f \sim 0.564$ to get the experimental value of $\mu_{\Omega^-} = (-\,2.019 \pm 0.054)\,\mu_N$ [11]. The $\mu_{\Delta^{++}}$ is known only approximately, $(6.14 \pm 0.51)\,\mu_N$ [1] and a better determination will enable us to pinpoint χ. As such the experimental determination is very important since it gives us a very large magnetic susceptibility χ.

Dahiya and Gupta [21], in their paper on decuplet μ_B, seem to be unaware of the 2001 publication of [1] and their fit to $\mu_{\Delta^{++}}$ is poor.

3 Results and discussion

We find that using the constrained values of the parameters a and b [14] one can get a good fit to the known decuplet magnetic moments. The moments may be used to

pinpoint (1) the susceptibility χ, (2) f and (3) ϕ, the ratio-s of the condensate and spin condensate for strange and ud quarks.

For octet magnetic moments two sum rules are written down from QCDSR [eqn.(4) and (5)]. These two sum rules are used to rule out some specific quark model calculation which claim to have fitted experimental magnetic moments satisfactorily but are obviously in contradiction with QCDSR. It is just that the sum rules highlight the discrepancies in the particular combination of the moments, to point out the inadequacy of the models. We hope future theoretical models will try to accommodate these new sum rules in their fitting while the new experimental data will continue to satisfy them.

It is interesting that the quark and gluon condensates can be written out directly in terms of octet magnetic moments [eqn.(2) and (3.)].

Finally we hope there will be more experimental data on baryon magnetic moments since it helps us to pinpoint QCD vacuum properties via QCDSR technique.

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